

**MTH 111**

**Midterm II**

**Spring 2012**

**Question 1.** Suppose that the value of a specific bond in the stock market is given by

$$f(t) = t^4 - 12t^3 + 16t^2 \quad (t \text{ in years})$$

where  $t = 0$  stands for the year 2010. Between the years [2010, 2016] when is the best time to

- a) purchase this bond
- b) sell this bond

**Question 1.** Find the absolute minimum and the absolute maximum of the function

$$f(x) = (x^2 + 2x)^3$$

on the closed interval  $[-1, 3]$ .

**Question 2.** A poster is to have an area of  $180 \text{ in}^2$  with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?

**Question 3.** Differentiate the following functions

a)  $y = e^{\sin(2x)} - \frac{x}{\cos(x)} + 1$

b)  $y = 3^{\tan(x)} - \ln(x^2 + x)$

c)  $y = \sqrt{\ln(x) - 7^x + 10}$

**Question 4.** Given the function  $f(x) = x^4 - 4x^3$ .

- a) Find the intervals of increase and decrease,
- b) Find the critical point(s),
- c) Discuss the concavity of the function,
- d) Find the inflection points,
- e) Sketch the graph of  $f(x)$ .

**Question 4.** Sketch the graph of a function  $f(x)$  satisfying the following properties

- $\lim_{x \rightarrow 1^+} f(x) = \infty$ ,  $\lim_{x \rightarrow 1^-} f(x) = -\infty$
- $\lim_{x \rightarrow \infty} f(x) = 0$
- $\lim_{x \rightarrow -\infty} f(x) = 4$
- $f(0) = 0$

**Question 6.**

a)  $\int \frac{2x^4 - 4x^2 + 1}{x^3} dx$

b)  $\int \cos(x)e^{\sin(x)} dx$

c) Find  $f(x)$  if  $f'(x) = 2 - e^x - 2\cos(x)$  and  $f(0) = 2$

**Question 7.** If  $f(x)$  is a continuous function on an interval  $[a, b]$ , then

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} f(x_i)\Delta(x)$$

where

$$\Delta(x) = \frac{b-a}{n}, \quad x_i = a + i\Delta(x)$$

a) Explain briefly the formula above

b) Evaluate the integral  $\int_0^3 (x^2 - 2x + 10)dx$  using Riemann sums.

**Bonus.** If  $\int_0^{12} f(x)dx = 10$ , then what is  $\int_0^4 f(3x)dx = ?$